

Probability Theory Review: 2-2

- Conditional Independence
- How to derive Bayes Theorem based
- Law of Total Probability
- Bayes Theorem in Practice

Working with data in python



= Refer to python notebook

Random Variables, Revisited

X : A mapping from Ω to \mathbb{R} that describes the question we care about in practice.

X is a *continuous random variable* if it can take on an infinite number of values between any two given values.

X is a *discrete random variable* if it takes only a countable number of values.

Random Variables, Revisited

X : A mapping from Ω to \mathbb{R} that describes the question we care about in practice.

Example: $\Omega = \text{inches of snowfall} = [0, \infty) \subseteq \mathbb{R}$

X is a *continuous random variable* if it can take on an infinite number of values between any two given values.

X amount of inches in a snowstorm

$$X(\omega) = \omega$$

$$\mathbf{P}(X = i) := 0, \text{ for all } i \in \Omega$$

(probability of receiving exactly i inches of snowfall is zero)

What is the probability we receive (at least) a inches?

$$\mathbf{P}(X \geq a) := \mathbf{P}(\{\omega : X(\omega) \geq a\})$$

What is the probability we receive between a and b inches?

$$\mathbf{P}(a \leq X \leq b) := \mathbf{P}(\{\omega : a \leq X(\omega) \leq b\})$$

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How to model?

s?

inches?

Continuous Random Variables



Discretize them!
(group into discrete bins)

How to model?

Continuous Random Variables



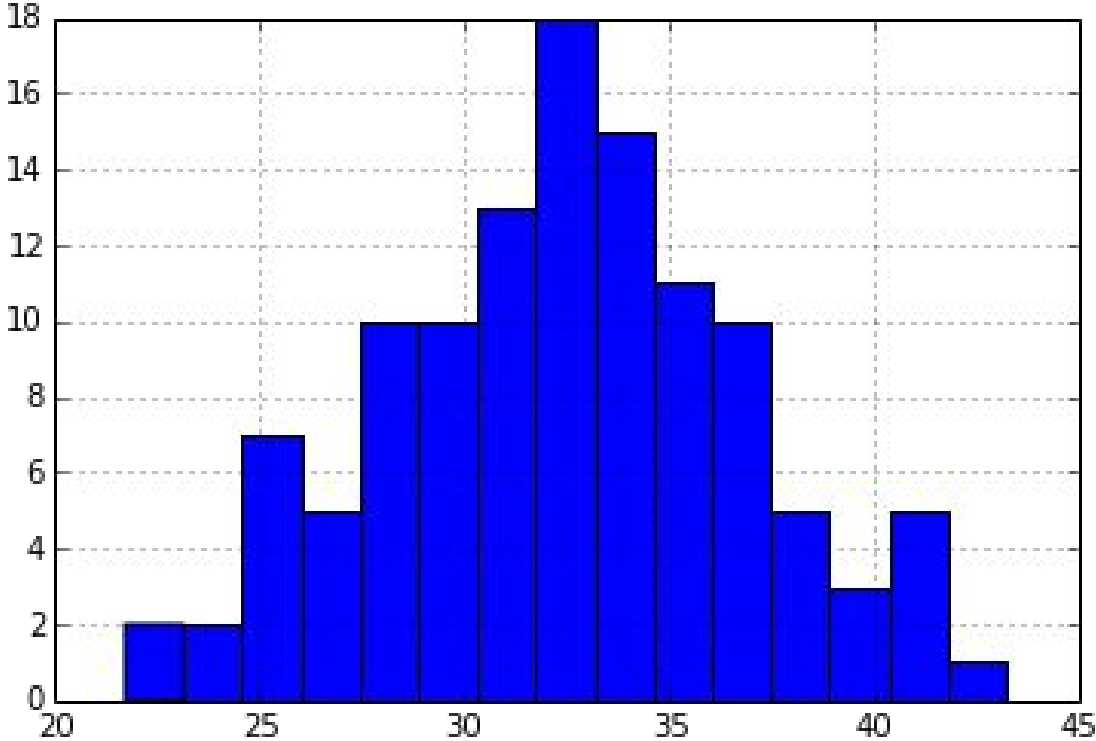
Discretize them!
(group into discrete bins)

How to model?



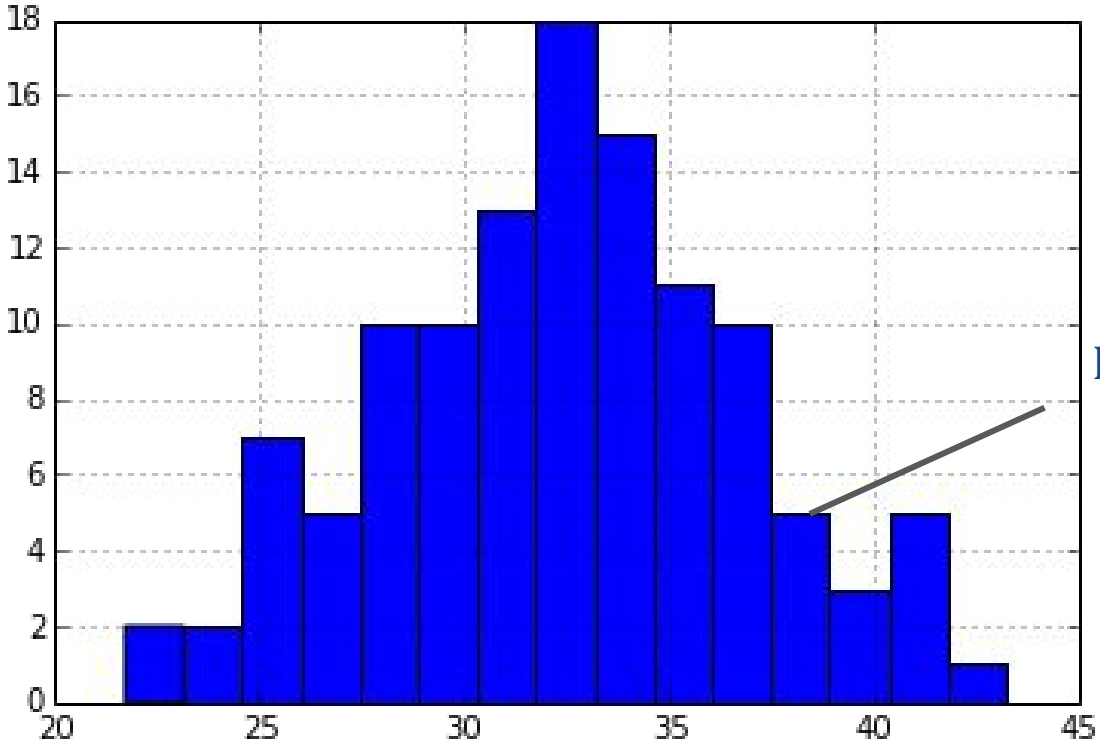
Histograms

Continuous Random Variables



Continuous Random Variables

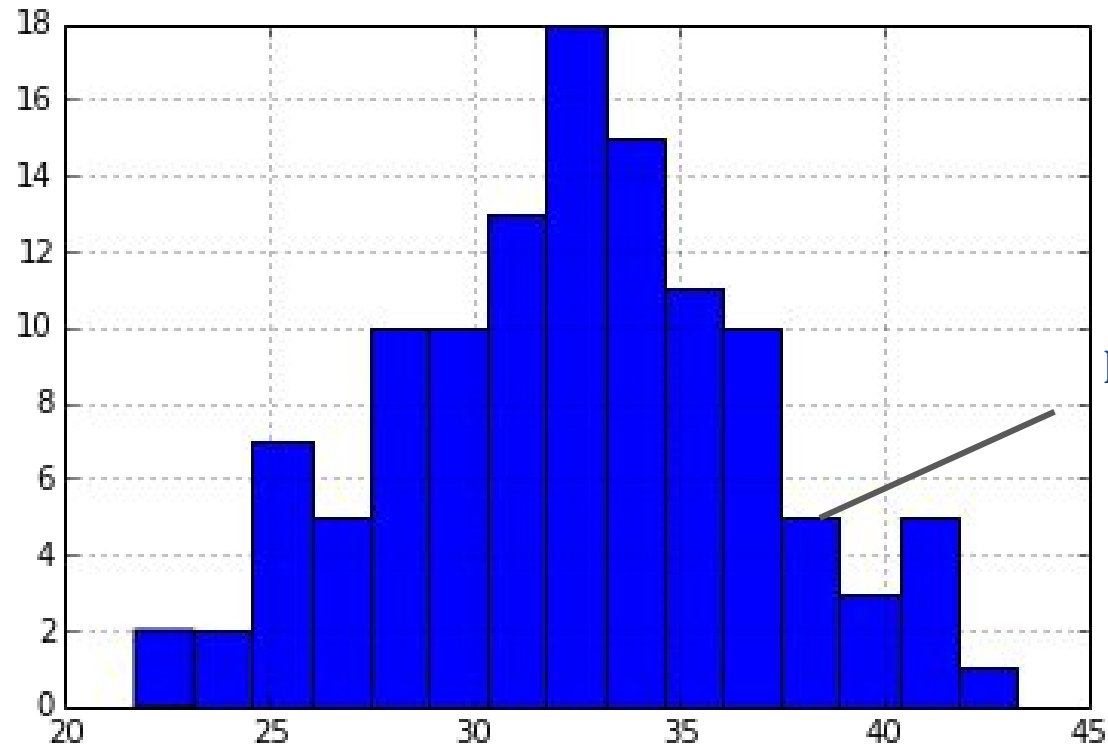
$$P(\text{bin}=8) = .32$$



$$P(\text{bin}=12) = .08$$

Continuous Random Variables

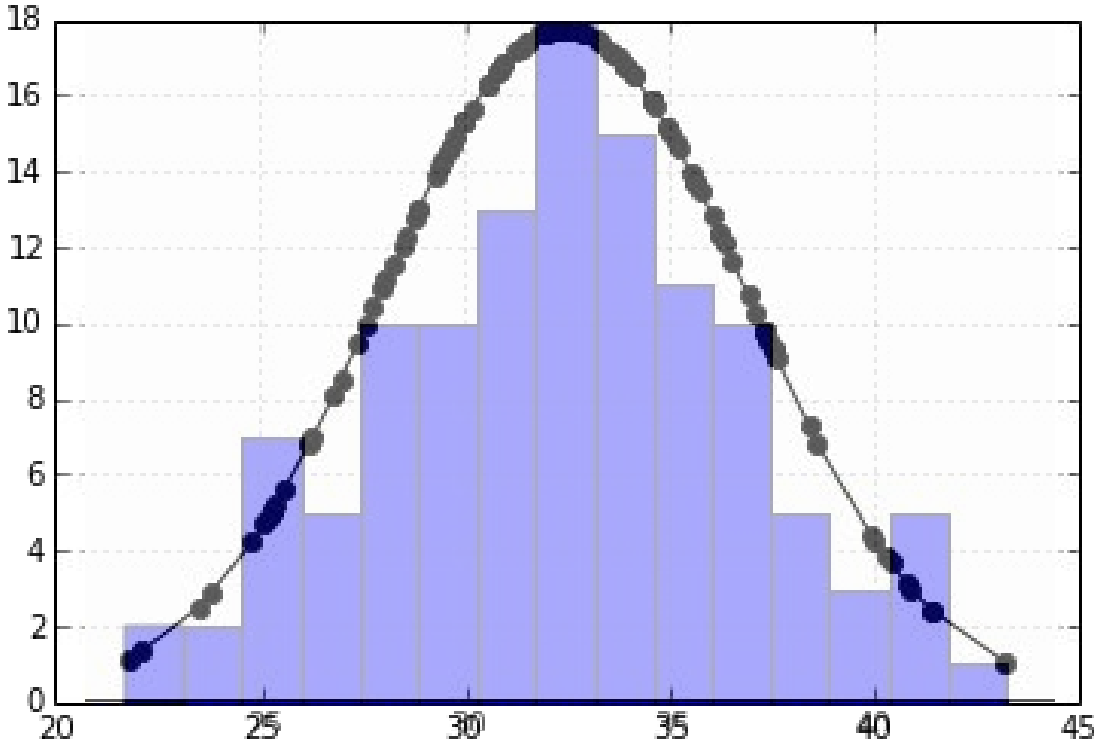
$$P(\text{bin}=8) = .32$$



$$P(\text{bin}=12) = .08$$

But aren't we throwing away information?

Continuous Random Variables



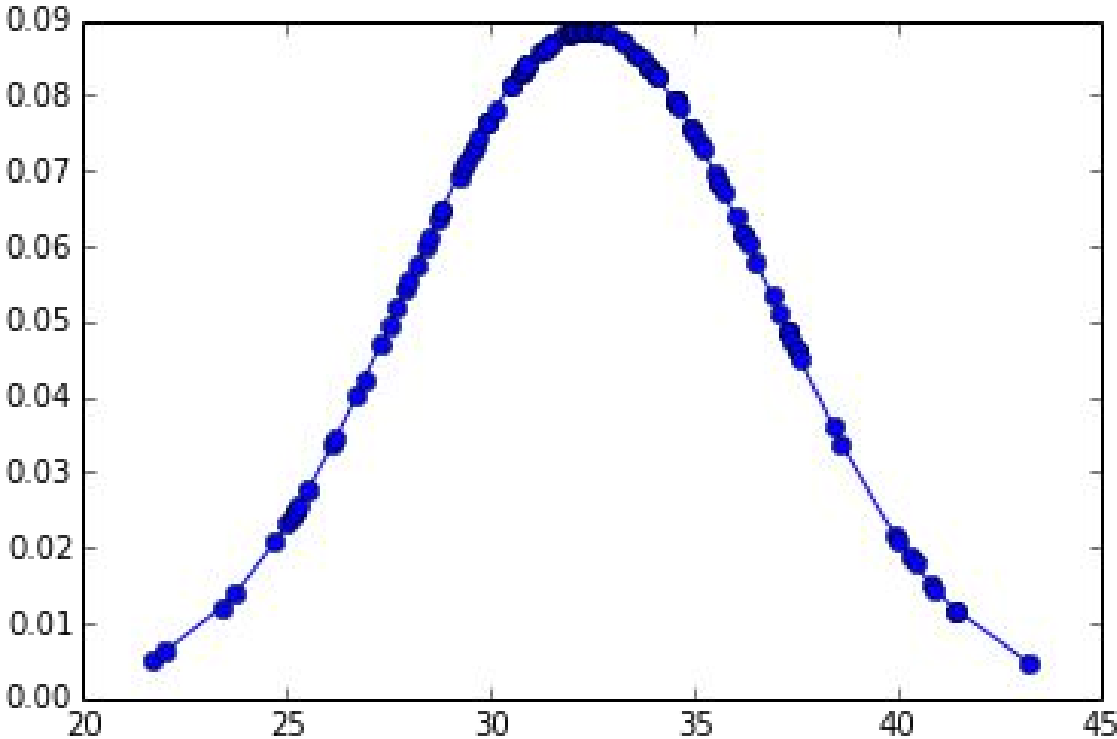
Continuous Random Variables

***X* is a *continuous random variable* if it can take on an infinite number of values between any two given values.**

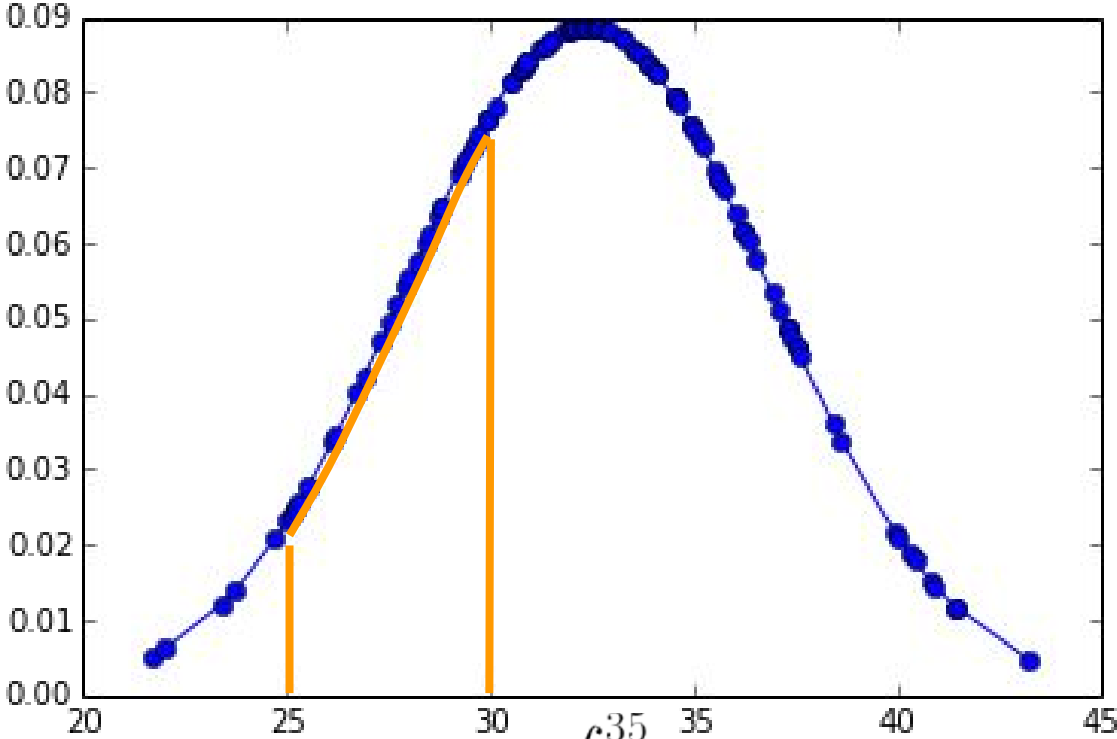
X is a continuous random variable if there exists a function $f_X(x)$ such that:

$$f_X(x) \geq 0, \text{ for all } x,$$
$$\int_{-\infty}^{\infty} f_X(x) dx = 1, \text{ and}$$
$$P(a < X < b) = \int_a^b f_X(x) dx$$

Continuous Random Variables



Continuous Random Variables



$$P(25 < X < 35) = \int_{25}^{35} f_x(x) dx$$